

Homework Assignment 2

ME7752: Introduction to Robotics

Due wednesday, Sep 30.

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Some of the following homework problems are taken from the exercises at the end of Chapter 2 of the text book (pages 54-61, appropriate edition). Where appropriate, you may write a MATLAB program and/or use the symbolic toolbox in MATLAB. If you use MATLAB, please state so. Other homework guidelines apply as usual. Most importantly, please follow the dropbox upload guidelines.

Total points = 39.

Q1. 3D rotations do not “commute” in general [9pts]. This is just Q2.11 from the book, embellished. “Commutativity” is the property relating to whether the order in which an operation is performed affects the results. For instance, addition and multiplication are commutative, whereas subtraction and division are typically not commutative.

a) Show that the product of two rotation matrices does not in general commute by giving an example. Make up two rotation matrices R_1 and R_2 , each about some fixed axis and by some angle, such that $R_1R_2 \neq R_2R_1$. We want a *numerical* example. [3pts]

b) Can you think of a simple (however trivial sounding) situation in which R_1 and R_2 do commute? That is, $R_1R_2 = R_2R_1$. Give a numerical example. [2pts]

c) Under what general conditions do two rotation matrices representing finite rotations commute? A proof is not required. [2 pts].

d) Say R_1 is rotation about the fixed X axis by $\pi/2$ and R_2 is rotation about the fixed Z axis by $\pi/2$. Show non-commutativity of these two rotations by performing them in sequence on your “textbook.” Illustrate your response by sketching appropriate pictures. Make sure you orient the reader by labeling your sketches of the book with “front”, “back”, etc. [2pts]

Q2. This is 2.4 from book, repeated below and slightly embellished [10pts]:

A frame $\{B\}$ is located initially coincident with a frame $\{A\}$. We rotate $\{B\}$ about \hat{Z}_B by 30 degrees, and then we rotate the resulting frame about its current \hat{X}_B by 45 degrees, giving the final reference frame $\{B\}$.

a) Give the rotation matrix that will change the description of vectors from ${}^B P$ to ${}^A P$. [5pts]

b) If *in addition* to the above two rotations, there was a translation of 5 units along \hat{X}_A and 1 unit along \hat{Z}_A to obtain the final position of $\{B\}$, what is the homogeneous transform ${}^A_B T$. What is ${}^B_A T$? [5 pts]

Q3. This is 2.13 from book, repeated below. You may use MATLAB if necessary. [5pts]

Say the following transformation relations between the frames $\{U\}$, $\{A\}$, $\{B\}$, and $\{C\}$ are known. Determine ${}^C_B T$.

$${}^U_A T = \begin{bmatrix} 0.866 & -0.5 & 0 & 11 \\ 0.5 & 0.866 & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$${}^A_B T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.866 & -0.5 & 10 \\ 0 & 0.5 & 0.866 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$${}^U_C T = \begin{bmatrix} 0.866 & -0.5 & 0 & -3 \\ 0.433 & 0.75 & -0.5 & -3 \\ 0.250 & 0.433 & 0.866 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Q4 [10pts]. This is a MATLAB animation exercise that takes advantage of your expertise in the representation of rotations in terms of rotation matrices.

First, draw a “cube” in MATLAB, centered about the origin, each edge of length 2, and one of the flat faces facing up. A globally fixed frame $\{A\}$ is defined, and its axes are aligned with MATLAB’s X, Y, and Z axes. The cube is drawn by drawing the twelve line segments corresponding to the cube’s edges. The beginnings of the program, namely drawing some edges of the cube, are provided below. Run this code fragment to see what it does. You are free to use this code fragment or ignore it.

```
% MATLAB code fragment
% drawing the cube of side 2, in 3D.
clear all; close all;
figure(1);
% the eight vertices of the cube
figure(1);
% the eight vertices of the cube
% 4 vertices on top face
V1 = [1;1;1]; V2 = [-1;1;1]; V3 = [-1;-1;1]; V4 = [1;-1;1];
% 4 vertices on bottom face
V5 = [1;1;-1]; V6 = [-1;1;-1]; V7 = [-1;-1;-1]; V8 = [1;-1;-1];
% 4 top edges
e1 = plot3([V1(1) V2(1)], [V1(2) V2(2)], [V1(3) V2(3)], 'r');
hold on; axis equal; xlabel('x'); ylabel('y'); zlabel('z');
axis([-2 2 -2 2 -2 2]);
e2 = plot3([V2(1) V3(1)], [V2(2) V3(2)], [V2(3) V3(3)], 'k');
e3 = plot3([V3(1) V4(1)], [V3(2) V4(2)], [V3(3) V4(3)], 'b');
e4 = plot3([V4(1) V1(1)], [V4(2) V1(2)], [V4(3) V1(3)], 'm');
% 4 bottom edges
e5 = plot3([V5(1) V6(1)], [V5(2) V6(2)], [V5(3) V6(3)], 'r');
e6 = plot3([V6(1) V7(1)], [V6(2) V7(2)], [V6(3) V7(3)], 'k');
...
```

If you wish to make the cube’s faces colored and opaque, you may check out the `patch` command.

Write a MATLAB program that will gradually rotate the cube about the \hat{Z} axis for a few seconds. And then the cube starts moving upward (along the \hat{Z} axis) gradually as it still rotates about the \hat{Z} axis.

Q5. [5 pts] Using the axis-angle representation of the rotation matrix, slightly modify the program from Q4, to spin the cube about an axis in the direction $[1 \ 1 \ 1]$ i.e., the long diagonal of the cube passing through the origin.

Look up the expression for the rotation matrix given an ‘axis-angle representation’ of a rotation.

Q6 [6 pts]. Figure 1 shows a triangular mechanism. Points O and B are fixed to a table with pin joints (revolute joints). The segments OA and AB have linear actuators so that their lengths L_1 and L_2 can be

changed as necessary. Point B is fixed at $(1, 0)$.

a) Say the range of motion on the linear actuators are such that $1 \leq L_1 \leq 2$ and $1 \leq L_2 \leq 2$. For this actuator range of motion, determine the reachable workspace of the end-point A. (That is, set of all points reachable by A.)

b) Now, assume that there is no bound on the range of motion of L_1 and L_2 . Make an animation of the mechanism as the point A tracks arbitrary user-specified curves. You may modify the code I have uploaded for the two-link 'serial' manipulator, in which we tried to make the end-point track a spline curve specified by the user by clicking on a bunch of points. (In a later HW, we might make a simple physical version of this robot perform this task.)

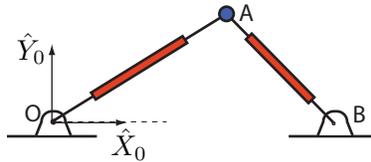


Figure 1: A triangular mechanism, with two linear actuators, OA and AB, which can change lengths L_1 and L_2 .