Homework 3
ME7752, Fall 2015

Instructor : Prof. Manoj Srinivasan
Due Oct 16, 2015

Total = 11+3+13+7+5 = 39 points.

Q1. Degrees of freedom. [11 pts] Please draw pictures or explain in words how you counted the degrees of freedom (DOFs).

1. A rigid body in 3D, unconstrained, has 6 DOFs. How many do a pair of scissors, again unconstrained in 3D, unconstrained, have? [2 pts]

2. Imagine a book with 100 individual sheets of paper (this includes the two covers). Imagine this book to be floating in 3D space, unconstrained. How many degrees of freedom does the book have? Assume that all pages are rigid and are hinged at the book's spine (which is a separate rigid body). [2pts]

3. Close your fingers in a fist, so the fingers cannot move independently (that is, neglect the degrees of freedom in your fingers). Now, starting from the shoulder joint, how many degrees of freedom does the human arm have? [2pts]

4. Go to the Computer Room E225 on the second floor of Scott Lab. How many degrees of freedom do the HP computer monitors in this room have, if you assume that the base of the monitor is rigidly bolted to the table? What if the monitor can slide horizontally on the table, but cannot lift off it? [2pts]

5. How many degrees of freedom does a piece of string have? What about a snake? Snake-like robots (or elephant trunks or octopus tentacles) are called **hyper-redundant robots**, because they seem to have many more DOFs than are required for the tasks they perform. Can you imagine scenarios in which one might need so many degrees of freedom? [2pts]

6. A car has lots of moving parts, so lots of degrees of freedom. But blurring your vision a bit and looking from above the car, you can more or less describe a car's movement in the following way: can move forward and back, can turn, but cannot move sideways. People (some mathematically-oriented dynamics researchers) thus model a car most simply as a little line segment as seen from above (a 'skate' they call it), capable of moving along the direction of the line segment and also turning/rotating as it moves. So instantaneously it has two degrees of freedom (one translation, one rotation); instantaneously, it cannot move sideways, perpendicular to the little line segment. But as we all know, even though a car cannot move sideways instantaneously, it can achieve a sideways displacement by some mixture of going forward-back and turning; we call this a parallel parking maneuver. Thus, this idealized car, even though it has only **two** "instantaneous" degrees of freedom has a three dimensional space of accessible configurations in the 2D plane – it can be in any position in any orientation. That is, to describe the position and orientation of a car in a plane, it takes **three** independent parameters (x, y, and θ). [0 pts] [1]

[1] I don't have a question here. I just wanted to introduce you to this weirdness. It turns out this is a new special type of system, called a "non-holonomic system", for which two ways of defining degrees of freedom do not coincide. These systems are perfectly physical and do not constitute a contradiction or anything; just unconventional. If you are lucky, you might encounter such systems in an advanced mechanics or controls course.

Q2. Typing on a keyboard. [3pts] What is the *minimum* number of degrees of freedom do you need in a robot arm to be able to type on a keyboard, one key at a time (that is, ignore Shift, Control keys, etc)?

The reasoning is similar to many general-purpose industrial robots have six degrees of freedom: because one general task is to manipulate an object in 3D — control its position and orientation, which is a 6 DOF task and therefore requires 6 DOF robot (for instance, look up the Adept Viper robot and the Kuka Industrial robot online). Typically, we match the number of robot DOFs to the number of task DOFs.

Q3. 3D Kinematics of a 3D manipulator [13 pts]. Consider the 3 link, non-planar RRR manipulator discussed in the lectures, shown in Fig. 1 below. We derived the DH parameters and defined a set of body-fixed axes for each link, including link {0}, the robot base. We also derived an expression for the position of a point P on the end-effector, represented in the base frame.

Say the origins of the respective frames are O_1, O_2, O_3, etc. Also, $L_1 = 0.1$, $L_2 = 1$, $L_3 = 0.1$, $L_4 = 0.95$.

(1) Write a `MATLAB` function that computes the end-point position P in frame {0}, namely ${}^0P$, given the joint angles θ_1, θ_2, θ_3. Using this program, find the end-point position when $\theta_1 = 0,\ \theta_2 = \pi/2,\ \theta_3 = 0$. [6pts]

(2) Either using the above function, or by other reasoning, characterize the reachable workspace of the end point P, if the joint angles all have 360 deg ranges of motion. Try to describe in words what the reachable workspace looks like. [7pts]

Q4. 3D animation. Say you wanted to do an animation of this robot, given $\theta_1(t)$, $\theta_2(t)$ and $\theta_3(t)$.

Explain in words how you would accomplish this animation. We did a version of this in class – you may use the same ideas. For $t = 0$ to 5, make an animation (or modify what we used in class) when $\theta_1(t) = \sin t$, $\theta_2(t) = \cos 2t$, and $\theta_2(t) = \sin t + \cos 2t$. [5 pts].

Q5. Inverse kinematics [7pts]. Find as many different values for the θ_i's if the desired end-point position is $(-1, 1, 0.5)$. Use `fsolve` in MATLAB to do this and use many different initial seeds to possibly converge to different solutions. Make sure to the angles obtained to some standard range, like 0 to 2π or $-\pi$ to π. This helps your not counting the same solution twice.

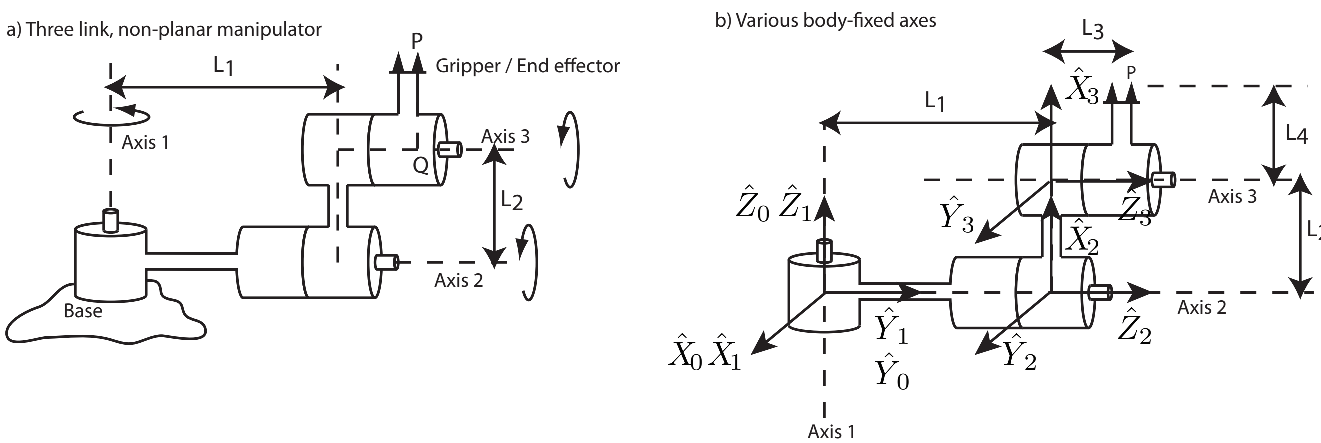

Figure 1: Three-link, non-planar manipulator, with 3 revolute joints.