Homework 5. ME 7752, Robotics, Fall 2015
Robot Dynamics in 2D
Due date: See your email.

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All usual homework guidelines apply. Please upload programs on Carmen on compressed folders. Total = 32 points.

Dynamics in 2D of a 3 link arm

Q1. ODE solution practice. Using ode45, solve the differential equation $m\ddot{x} + kx + c\dot{x} = 0$ for $x(0) = 1$ and $\dot{x}(0) = -1$. Plot $x(t)$ for time $t = 0$ to 5. $m = 1$, $c = 8$, $k = 4$.

Q2a. Consider the general planar three link arm, analogous to the two link planar arm from the lectures. Derive the dynamics equations of motion for this manipulator using MATLAB. The equations will in general involve all or some of m_1, m_2, m_3, I_1, I_2, I_3, g, r_1, r_2, r_3, L_1, L_2, and L_3, where these quantities have meanings analogous to those used in the lecture. Please explain in words and/or equations the mathematical procedure for deriving the equations of motion. No need to actually write out the final equations by hand in your hard-copy, as they are too long. [8 points]

Q2b. Using MATLAB, write a program to integrate the differential equations for the two link manipulator, setting the torques identically to zero, but including gravity. Assume $L_{1,2,3} = 1$, $m_{1,2,3} = 1$, the center of masses are at the center of the links, and $I_{1,2} = 1/12$. Plot $\theta_1(t)$, $\theta_2(t)$, and $\theta_3(t)$ for $t = 0$ to 5, given the initial conditions $\theta_1(0) = \pi/4$, $\theta_2(0) = 0$, $\theta_3(0) = 0$, $\dot{\theta}_1(0) = 0$, $\dot{\theta}_2(0) = 0$, $\dot{\theta}_3(0) = 0$. [10 points]

Q2c. Given that the system has no damping or energy input, show that your simulation is (likely to be) correct by computing the total energy (kinetic + potential) $E(t)$ as a function of time and noting that this is essentially a constant in time (check that the variation is less than 10^{-7} or so). [6 points]

Q2d. Pick an initial conditions that gives you highly non-periodic "chaotic" looking non-periodic trajectories. Then, start the simulation from two very close initial conditions in this region, different by only about 10^{-5} in one of the state variables. Plot, say $\theta_1(t)$ for these two neighboring (very close) initial conditions for a sufficiently long time to show how they soon diverge and become essentially unrelated movements (choose time duration appropriately to show the divergence). [4 points]

Q3. Inverse dynamics. Answer in words (we will make this quantitative in the next HW). For the above system, if you are given the angles $\theta_1(t)$, $\theta_2(t)$, and $\theta_3(t)$ as continuous functions of time, how will you find the torques $\tau_1(t)$, $\tau_2(t)$ and $\tau_3(t)$? [4 points]