

## Optimal speeds for walking and running, and walking on a moving walkway

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Many aspects of steady human locomotion are thought to be constrained by a tendency to minimize the expenditure of metabolic cost. This paper has three parts related to the theme of energetic optimality: (1) a brief review of energetic optimality in legged locomotion, (2) an examination of the notion of optimal locomotion speed, and (3) an analysis of walking on moving walkways, such as those found in some airports. First, I describe two possible connotations of the term “optimal locomotion speed:” that which minimizes the *total* metabolic cost per unit distance and that which minimizes the *net* cost per unit distance (total minus resting cost). Minimizing the total cost per distance gives the maximum range speed and is a much better predictor of the speeds at which people and horses prefer to walk naturally. Minimizing the net cost per distance is equivalent to minimizing the total daily energy intake given an idealized modern lifestyle that requires one to walk a given distance every day—but it is not a good predictor of animals’ walking speeds. Next, I critique the notion that there is no energy-optimal speed for running, making use of some recent experiments and a review of past literature. Finally, I consider the problem of predicting the speeds at which people walk on moving walkways—such as those found in some airports. I present two substantially different theories to make predictions. The first theory, minimizing total energy per distance, predicts that for a range of low walkway speeds, the optimal absolute speed of travel will be greater—but the speed relative to the walkway smaller—than the optimal walking speed on stationary ground. At higher walkway speeds, this theory predicts that the person will stand still. The second theory is based on the assumption that the human optimally reconciles the sensory conflict between the forward speed that the eye sees and the walking speed that the legs feel and tries to equate the best estimate of the forward speed to the naturally preferred speed. This sensory conflict theory also predicts that people would walk slower than usual relative to the walkway yet move faster than usual relative to the ground. These predictions agree qualitatively with available experimental observations, but there are quantitative differences. © 2009 American Institute of Physics. [DOI: [10.1063/1.3141428](https://doi.org/10.1063/1.3141428)]

**A popular hypothesis in legged locomotion is that we move in a manner that roughly minimizes the energy cost. A brief account of the evidence for and against this hypothesis is presented. A specific instance of possible energy minimization is the preference of specific walking speeds by humans when walking in a manner that is most comfortable. What energy-like quantity should humans minimize? Should humans minimize the total energy cost per unit distance walked or should they minimize the energy cost per distance over and above what they expend while resting? Some adaptationist evolutionary arguments and experimental evidence favor the minimization of the total cost. Unlike for walking, there is a common perception that running has no optimal speed. We critique this perception, building on the discussion on optimal walking speeds and drawing from past literature. Finally, the problem of walking on moving walkways found in some airports is considered. Two theories are presented: one based on energy minimization and another based on the idea that the brain may get confused be-**

**cause the walking speed the eyes see is different from the walking speed the legs feel. For low walkway speeds, both theories predict that people would walk slower than usual relative to the walkway yet move faster than usual relative to the ground. At higher walkway speeds, the energy minimization theory predicts that people would ride the walkway standing still.**

### I. INTRODUCTION

Why do humans and other animals walk and run the way they do? This is an ancient question, dating back at least to Borelli, a contemporary of Newton. Borelli<sup>1</sup> provided a candidate answer. Paraphrasing Borelli, perhaps we move in a manner that minimizes effort or some energy cost given some caveats. In this paper, a brief review of this energy minimization hypothesis is presented, including some evidence for and against it (Sec. II).

A specific special case of energetic optimality is the notion of the “optimal locomotion speed” and how this speed compares with the speeds people use when they walk naturally and unhurried. Different definitions of the term optimal speed are possible depending on what quantity is minimized,

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leading to different optimal speeds. Which of these different optimal speeds is most appropriate? Here, I present idealized scenarios for which each of these definitions of optimal speed is appropriate (Sec. III B) and recall that minimization of the total metabolic cost per unit distance agrees well with the preferred walking speeds. Next, the notion of the optimal running speed is considered (Sec. III D). It is sometimes noted that there is no optimal running speed in that all running speeds are equally good from the perspective of energy minimization. This notion is critically examined here, drawing from past literature and recently published experimental results.

Finally, I consider the problem of walking on the moving walkways found in some airports. Can we predict the speeds at which people would walk given the speed of the moving walkway? This question is examined in Sec. IV using two different theories, one based on energy optimality and another based on the observation that there might be a conflict between the sensory information from the eyes and the legs. We find that both these theories have qualitatively similar predictions at slow walkway speeds but are different quantitatively. Both these theories agree qualitatively with available observations of people walking on moving walkways.

## II. BACKGROUND: A BRIEF REVIEW OF ENERGY OPTIMALITY

The best way to locomote depends on the situation. High accelerations and maneuverability may be required to successfully catch an evading prey or to successfully evade a pursuing predator (e.g., Ref. 2). At other times, the ability to maintain a slow but sufficient speed for long periods of time (endurance) might be important (e.g., Ref. 2). Not wasting energy—energetic economy—closely related to endurance, is useful in most situations. Not losing stability—not falling down and not getting knocked wildly off course—is usually a constraint in all these situations (e.g., Ref. 3). These are not necessarily mutually exclusive goals. For example, an animal can accelerate quickly when it wants to but be energetically economical when it has reached the intended steady speed. Energetic optimality could be pursued, for example, with stability as a constraint.

Energetic optimality has been one of the more successful predictive theories of the observed features of legged locomotion, especially for larger animals. Unlike optimality with respect to some as-yet-undetermined criterion, energetic optimality is an experimentally refutable hypothesis. Various studies have found some evidence for energetic optimality in animal movement. This evidence is of two types:

(1) Direct experimental evidence. The metabolic energy expenditure during a naturally preferred locomotion pattern is compared with those of nearby not-preferred and uncommon locomotion patterns. These experiments usually find that the preferred locomotion pattern tends to have the lower metabolic cost. In these experiments, the metabolic cost here is typically estimated by monitoring the oxygen ( $\dot{V}_{O_2}$ ) and carbon dioxide ( $\dot{V}_{CO_2}$ ) exchange through the lungs (e.g., Ref. 4).

(2) Consequences of optimality. This kind of evidence is less direct. A researcher uses a mechanistic mathematical model of the animal to determine the consequences of optimality for the model and compares that to observed behavior.

Before discussing the experimental evidence of the first type, we need to more clearly define what we mean by “animals pursue energetic optimality.” When an animal is resting—that is, not moving—it expends metabolic energy to support the various physiological processes that keep it alive. While “not moving” is not a single physiological state, it is convenient to assume this resting metabolic rate (energy per unit time) to be a constant  $e_{rest}$ . When the animal starts moving, voluntary muscles are employed to perform work on the body and exert forces as appropriate, increasing the metabolic rate. We represent the *total* rate of metabolic energy expenditure during locomotion as  $\dot{E}_m$ . The *net* metabolic rate during locomotion—that is, the metabolic rate during locomotion over and above the resting rate—is given by  $\dot{E}_m^{net} = \dot{E}_m - e_{rest}$ . If the animal is moving at an average speed  $v$ , the total metabolic energy per unit distance is  $E_1 = \dot{E}_m/v$  and the net metabolic cost per unit distance is  $E_2 = (\dot{E}_m - e_{rest})/v$ . Sometimes, the total cost per unit distance is nondimensionalized by dividing by the weight  $mg$  of the animal and is referred to as *the cost of transport*.

The values of these various energy-related quantities, namely,  $\dot{E}_m$ ,  $\dot{E}_m^{net}$ ,  $E_1$ , and  $E_2$ , depend on the various variables that characterize how the animal is moving: the speed  $v$ , the stride length, and eventually, the detailed time-dependent ways in which the various muscles are used and the various limbs are moved. The term “energy optimality” usually means that the details of how the animal moves are selected in a manner that minimizes one of these quantities ( $\dot{E}_m$ ,  $\dot{E}_m^{net}$ ,  $E_1$ , and  $E_2$ ).

Which of these various quantities should animals and humans minimize? If the average forward speed  $v$  is constrained and therefore a constant, as in a treadmill, minimizing  $\dot{E}_m$ ,  $\dot{E}_m^{net}$ ,  $E_1$ , and  $E_2$  are all equivalent, the four quantities being linearly related to each other with positive coefficients. In the next section (Sec. III), we consider in some detail walking and running when the speed  $v$  is not constrained but can be self-selected when minimizing these three quantities gives different optimal behaviors.

Now, we turn to some specific experimental evidence for energy optimality. As just noted, we will consider the energetic optimality of the preferred locomotion speeds in Sec. III. For any given speed, humans appear to select the stride length, or equivalently the stride frequency, that seems to minimize the metabolic cost.<sup>5–8</sup> Similarly, humans also seem to choose their step width based on energetic considerations.<sup>9</sup> That is, stride widths and stride lengths smaller or larger than what humans naturally prefer tend to require greater metabolic expenditure, as estimated by measuring the oxygen exchange  $\dot{V}_{O_2}$ .

In humans, gait transition from walking to running or running to walking happens close to where energetic costs of

walking and running are approximately equal,<sup>10–13</sup> as would be consistent with energetic optimality. However, some of these studies show that there is a small but significant difference between the speed at which humans prefer to change gait and the speed at which the walking and running cost curves cross—also they find some differences in the walk-run and the run-walk transition speeds (e.g., Ref. 10). This small difference, depending on one’s perspective, may be taken as evidence against energetic optimality as being an accurate-enough theory, at least for gait transitions on treadmills. Similarly, there is some conflicting evidence regarding the trot-to-gallop transition in horses. While one study<sup>14</sup> found a statistically significant difference between the minimum sustained galloping speed and the energetically optimal trot-to-gallop transition speed, another study<sup>15</sup> found no significant difference.

When humans walk, their hips travel in an up and down motion, as they would if they were vaulting over on a relatively straight leg like an inverted pendulum.<sup>16,17</sup> Experiments that required the human subjects to walk in a manner that reduced this up and down hip motion found increased energy expenditure.<sup>18,19</sup>

Finally, humans also seem to be able to do energy optimization dynamically under novel constraints as the situation demands it. For instance, the different relationships between speed and step length under different constraints (constrained speed, step length, or step frequency) seem to be roughly consistent with the optimization of the metabolic cost subject to these specific constraints.<sup>5</sup> However, more experiments are needed to more thoroughly demonstrate the natural constrained optimization capabilities of humans in strange and unfamiliar situations.

The experiments described above provide evidence that some aspects of animal behavior might be a consequence of, or at least correlated with, energetic optimality. However, the experiments described above were narrow in scope. They do not say that among all the possible gaits that our legs are capable of, we choose the best possible. To show this more general claim, we have to compare the energetic cost of the preferred gait with every other possible gait. This takes far too many experiments. Moreover even if we did perform a large number of experiments and found that the self-selected gait has the lowest metabolic cost, the result would be subject to the criticism that strange and unpracticed gaits are energetically expensive exactly because they are strange and unfamiliar. Thus the more elaborate tests of energetic optimality might best be carried out with mathematical models. Such mathematical models, having assumed energetic optimality, should be able to make a variety of testable predictions about the locomotor behavior of animals. There have been a number of such studies, using models of varying complexity, that have been successful to various extents in predicting and explaining specific features of legged locomotion (e.g., Refs. 8, 12, 17, and 20–24). A detailed discussion of such studies is beyond the scope of this paper (see Ref. 25 for a more detailed review)—but we point out that at one extreme is the work of Anderson and Pandy,<sup>22</sup> which used a model of great detail and complexity, and at the other extreme is the work of Alexander and others, using much sim-

pler models,<sup>8,12,17,20,21,23</sup> some so simple as to have no biophysical parameters.<sup>17,24,25</sup> These two modeling extremes have been complementary in advancing our understanding of the role of energetic optimality in legged locomotion. While the detailed models might allow us to predict the individual muscle forces in detail, the simpler models have been used to obtain a broad understanding of the basic principles—for instance, why humans choose to walk at low speeds and run at high speeds.<sup>12,17,20,24</sup>

### III. OPTIMAL LOCOMOTION SPEEDS

#### A. Preferred speeds

In a classic study, when Ralston<sup>26</sup> asked his healthy human subjects to walk at a speed that is most “natural” or “comfortable” to them (Ralston’s words), they consistently walked at about 1.2–1.3 ms<sup>-1</sup>. This speed is often called “the preferred speed” of walking, sometimes called the self-selected speed, here denoted by  $v_p$  (see also Refs. 27 and 28).

The definition of preferred speed derived from the experiment above is somewhat subjective given that it relies on the human subject’s interpretation of the experimenter’s instructions. The works of Pennycuik<sup>29</sup> and Hoyt and Taylor<sup>30</sup> provide a more objective way to define the preferred speed. Hoyt and Taylor,<sup>30</sup> for instance, observed horses moving freely on stationary ground for a sufficiently long period of time and plotted a frequency distribution of the walking and trotting speeds used by the horses. This frequency distribution was bimodal with two relatively distinct peaks, that is, the horses naturally favored specific walking and trotting speeds and preferred intermediate speeds less. The locations of these two peaks can be, relatively objectively, called the preferred walking speed and the preferred trotting speed, respectively. Similar observations were previously made by Pennycuik<sup>29</sup> for gnu walking and running in the wild.

Is it possible to predict these naturally preferred speeds from energetic optimality? This question is addressed next.

#### B. Optimal walking speed

Using  $\dot{V}_{O_2}$  measurements of people walking on treadmills at various constant speeds, Ralston and others (e.g., Refs. 26 and 31) estimated the metabolic rate  $\dot{E}_m$  as a function of walking speed  $v$ . They found this total metabolic rate data (average cost per unit time) to be increasing, curvilinear, and well approximated by the following empirical quadratic equation in speed  $v$ :

$$\dot{E}_m = a_1 + a_2 v^2. \quad (1)$$

If the resting cost is  $e_{\text{rest}}$ , the net locomotion cost per unit time is

$$\dot{E}_m^{\text{net}} = (a_1 - e_{\text{rest}}) + a_2 v^2. \quad (2)$$

Both the total metabolic rate  $\dot{E}_m$  and the net metabolic rate  $\dot{E}_m^{\text{net}}$  are monotonic increasing in  $v$  [Eqs. (1) and (2)]. So minimizing either of these metabolic rates (cost per unit time) requires walking as slowly as possible, that is, at in-

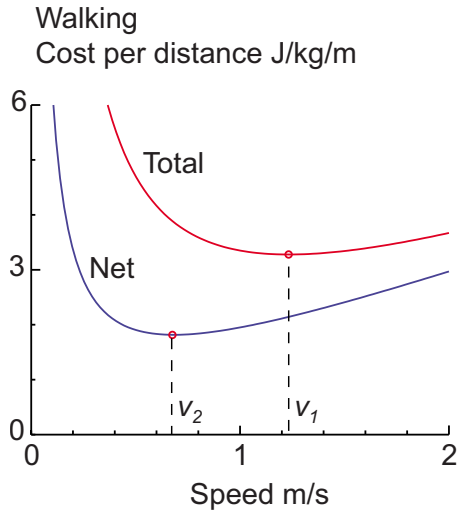


FIG. 1. (Color online) The total metabolic cost per unit distance and the net metabolic cost per unit distance (total cost minus the resting cost). The speed that minimizes the total cost is substantially higher than the net cost.

finitesimal speed. As is clear from Sec. III A, humans do not walk arbitrarily slowly even when there is no meaningful time constraint on the locomotion task.

Next, we consider the costs per unit distance. The total metabolic cost per unit distance  $E_1$  is given by

$$E_1 = \dot{E}_m/v = a_1/v + a_2v \quad (3)$$

and the net metabolic cost per unit distance  $E_2$  is given by

$$E_2 = (\dot{E}_m - e_{\text{rest}})/v = (a_1 - e_{\text{rest}})/v + a_2v, \quad (4)$$

as depicted in Fig. 1.

Using Eqs. (3) and (4), the optimal speeds  $v_1$  and  $v_2$  that minimize, respectively, the total cost per distance  $E_1$  and the net cost per distance  $E_2$  are obtained:

$$v_1 = \sqrt{a_1/a_2} \quad (5)$$

and

$$v_2 = \sqrt{(a_1 - e_{\text{rest}})/a_2}. \quad (6)$$

Note that  $a_1$  is the cost of walking at infinitesimal speed ( $v=0^+$ ), obtained by extrapolating the nonzero-speed metabolic data to zero speed—that is, using the regression equation Eq. (1). On the other hand,  $e_{\text{rest}}$  is the resting cost ( $v=0$ ). Usually,  $a_1 > e_{\text{rest}} > 0$ , so that we have  $v_1 > v_2 > 0$ .

For Ralston's subjects,<sup>26</sup> the constants roughly have the average values  $a_1=2.02$  W/kg and  $a_2=1.33$  W s<sup>2</sup>/kg m<sup>2</sup>, so that  $v_1 \approx 1.23$  ms<sup>-1</sup> (see Fig. 1). Using  $e_{\text{rest}}=1.4$  W/kg,<sup>31</sup> one obtains  $v_2 \approx 0.7$  ms<sup>-1</sup>.

When asked to walk at a comfortable and natural speed, Ralston's subjects chose to walk much closer to their  $v_1$  than  $v_2$ . For instance, the subject of Fig. 3 in Ref. 26 walked at about 1.22 ms<sup>-1</sup>.

Similarly, the two peaks of the speed distributions for Hoyt and Taylor's<sup>30</sup> horses aligned roughly with the  $v_1$  corresponding to the horses' walking and trotting gaits.

Preferred speeds of animals moving on slopes are different from self-selected speeds on level ground. At a range of slopes, the changes in preferred speeds are well predicted by the changes in the energetically optimal speed  $v_1$ , now computed using the metabolic cost for walking on slopes in horses<sup>32</sup> and in humans.<sup>28,33</sup> Preferred walking speeds of humans while carrying loads of up to 75% body weight matched the corresponding  $v_1$ .<sup>27</sup> Also, variously disabled human subjects walking on level ground, using crutches or prosthesis, seem to have preferred speeds that minimize their much higher metabolic costs per distance.<sup>26</sup> Thus, in a variety of situations, the minimization of the total metabolic cost per unit distance  $E_1$  seems to be a reasonably good predictor of the preferred walking speed, at least in comparison with the minimization of the net cost per distance  $E_2$ .

### C. Adaptationist accounts

What is the adaptive advantage of minimizing the total metabolic cost per unit distance  $E_1$ ? Researchers have argued that this speed  $v_1$  is the so-called *maximum range speed*, the speed which lets the animal travel the maximal distance for a given energy budget,<sup>34–36</sup> that is, on a single stomach full without eating. One might imagine that animals that need to travel long distances for food might use this speed  $v_1$ . Use of speeds close to  $v_1$  would be advantageous in a hunter gatherer. In times of food scarcity, when the probability of finding food increases monotonically with the distance traveled, the probability of starving to death is minimized by traveling at  $v_1$ . Or, if you ask a person to walk a given distance in a manner that leaves him with the greatest unused energy at the moment he completes the locomotion task—perhaps so that the unused energy could then be used for the performance of some other task—the person should walk at  $v_1$ . Minimizing  $E_1$  is analogous to maximizing mileage in automobiles. Thus the speed  $v_1$  makes good evolutionary sense.

Is there a plausible scenario in which it is appropriate to walk at the speed that minimizes the net metabolic cost per unit distance  $E_2$ ? Here I present one such. While humans may have had to travel long distances for food in their evolutionary past, they do not do so anymore. The following may be a more plausible “daily routine” for an idealized modern human, say, at a desk job or working on a farm. Over 1 day, equal to time duration  $T$ , we have the following:

- (1) He has to cover a certain average distance  $d$  every day by walking. If he travels at speed  $v$ , this takes time  $t_1 = d/v$ .
- (2) He spends a constant duration  $t_2$  of his time doing miscellaneous non-locomotion tasks, spending energy at some average rate  $e_2$ .
- (3) He spends the rest of his time  $t_3 = T - t_1 - t_2$  resting, spending energy at a rate  $e_{\text{rest}}$ .

The total energy cost over 1 day is given by

$$\begin{aligned}
E_{\text{day}} &= t_1 \cdot \dot{E}_m(v) + t_2 e_2 + t_3 e_{\text{rest}} \\
&= \frac{d}{v} \cdot \dot{E}_m(v) + t_2 e_2 + \left( T - \frac{d}{v} - t_2 \right) e_{\text{rest}} \\
&= \frac{d}{v} \cdot (\dot{E}_m(v) - e_{\text{rest}}) + \underbrace{t_2 e_2 + (T - t_2) e_{\text{rest}}}_{\text{a constant}} \\
&= d \cdot E_2 + \text{a constant}
\end{aligned} \tag{7}$$

This implies that minimizing the total energy cost per day,  $E_{\text{day}}$ , effectively minimizing food intake over a day, is equivalent to minimizing the net metabolic cost per unit distance  $E_2$ . The optimality of  $v_2$  in this context holds when  $T$  is equal to a time unit not equal to 1 day as long as  $T > d/v_2$ . If the time available  $T < d/v_2$ , then the speed that minimizes the total energy consumed over time  $T$  is  $d/T$ .

On the contrary, for this same modern human daily routine, note that the speed  $v_1$  would be appropriate if the human has a specific constant energy or food uptake per day,  $E_{\text{day}}$ , and would like to maximize the energy available ( $E_{\text{day}} - d/v \cdot \dot{E}_m$ ) for the performance of nonlocomotion tasks given that he must travel a specific distance  $d$  everyday.

As noted earlier, modern humans prefer to walk much closer to  $v_1$  than  $v_2$  when asked to travel at a speed that is most comfortable. Why? One might argue that old habits die hard, in a sense, and that no systematic evolutionary pressure has existed to adapt the preferred speed away from  $v_1$ , an apparent adaptation for a foraging lifestyle. Finally, we refer the reader to Hedenstrom and Alerstam<sup>37</sup> for a related account of optimal speeds for bird flight under a variety of situations.

#### D. Optimal running speed

In humans, optimal walking generally consumes less energy per unit distance than optimal running, so the optimal locomotion speed usually corresponds to a walking gait. Nevertheless it is meaningful to ask what the optimal speed would be if the gait was constrained to be running-like or restricted to speeds higher than about  $2.4 \text{ ms}^{-1}$  at which running is less expensive than walking.<sup>10,30</sup>

Classic work by Margaria *et al.*<sup>38</sup> suggested that the total metabolic cost per unit time, as estimated by  $\dot{V}_{\text{O}_2}$  measurements, was well approximated by a linear empirical relationship in speed  $v$ , that is,

$$\dot{E}_m = b_1 + b_2 v. \tag{8}$$

I use coefficients  $b_i$  in Eq. (8) to differentiate this expression from the walking cost rate from Eq. (1). Margaria *et al.* then found that  $b_1 \approx e_{\text{rest}}$ , so that the net cost per unit distance is  $E_2 = (b_1 - e_{\text{rest}})/v + b_2 \approx b_2$ , a constant. So they concluded that the (net) metabolic cost per unit distance was roughly independent of speed. This observation of the apparent speed independence of the cost per distance has permeated some of the locomotion literature, leading to the general belief that there is “no unique optimal speed for running.” See Ref. 39 for a recent review and refutation of this belief.

The apparent constancy of net cost per distance  $E_2$  has been argued as a potential advantage in a prehistoric hunter-gatherer context, perhaps giving early hominids some flexibility of speed (e.g., Ref. 40). Let us look at such arguments critically. First, as we have argued, minimizing the total cost per distance gives the maximum range speed  $v_1$ , and therefore possibly more appropriate in a foraging context than minimizing the net cost per distance. Similarly, when engaged in a persistent hunt—pursuing an animal until the animal tires to exhaustion—the total cost of the pursuit is likely the appropriate quantity to minimize, while ensuring that the prey is not lost; thus, again the net cost per distance is unlikely to be the quantity of interest.

Second, we might ask if the constancy of net cost per distance is well founded. While  $b_1$  might be close to  $e_{\text{rest}}$ , usually  $b_1$  (the cost of locomotion at infinitesimal speed) is slightly larger than  $e_{\text{rest}}$  (the cost for resting). Then, the net locomotion cost per unit distance  $E_2 = (b_1 - e_{\text{rest}})/v + b_2$  would be a monotonically decreasing function of  $v$ , giving an infinite optimal speed,  $v_2 = \infty$ , or more meaningfully, an optimal speed equal to the highest possible running speed. Similarly, the corresponding total cost per unit distance  $E_1 = b_1/v + b_2$  would also be minimized at the highest possible running speed. Of course, one does not expect the regression relation  $\dot{E}_m = b_1 + b_2 v$  to be valid at the highest running speeds, as the equation was derived from experiments at lower speeds. Nevertheless, it seems likely that neither  $E_1$  nor  $E_2$  are constant in detail with respect to speed variations.

More recently, Steudel-Numbers and Wall-Scheffler<sup>39</sup> showed that the total cost per unit time, again estimated by  $\dot{V}_{\text{O}_2}$  measurements (apparently more accurate than before), is better approximated by a curvilinear function, as opposed to the linear equation (8); for instance,  $\dot{E}_m = b_1 + b_2 v + b_3 v^2$ , with a small but positive  $b_3$ , making the function convex. This nonzero  $b_3$  makes it so that the total cost per distance  $E_1$  and the net cost per distance  $E_2$  are minimized at the following finite speeds:

$$v_1 = \sqrt{b_1/b_3} \quad \text{and} \quad v_2 = \sqrt{(b_1 - e_{\text{rest}})/b_3}.$$

It is clear from these equations that for small  $b_3$ , small changes in our estimation of  $b_3$  can mean large changes in the optimal speed ( $\partial v_{1,2}/\partial b_3 \rightarrow \infty$  as  $b_3 \rightarrow 0$ ). Despite this apparent sensitivity, Steudel-Numbers and Wall-Scheffler<sup>39</sup> managed to obtain reasonably consistent values for the optimal speed—about  $3.7 \text{ ms}^{-1}$  for males and about  $2.9 \text{ ms}^{-1}$  for females.

#### IV. WALKING ON MOVING WALKWAYS: TWO THEORIES

We now extend the discussions about optimal speeds and preferred speeds to a novel locomotion context. In many large airports, one finds “moving walkways”<sup>41</sup> that are in effect long treadmills or horizontal escalators and help transport people along long corridors. At what speeds do people naturally walk on these moving walkways? We consider two distinct theories for predicting this speed choice.

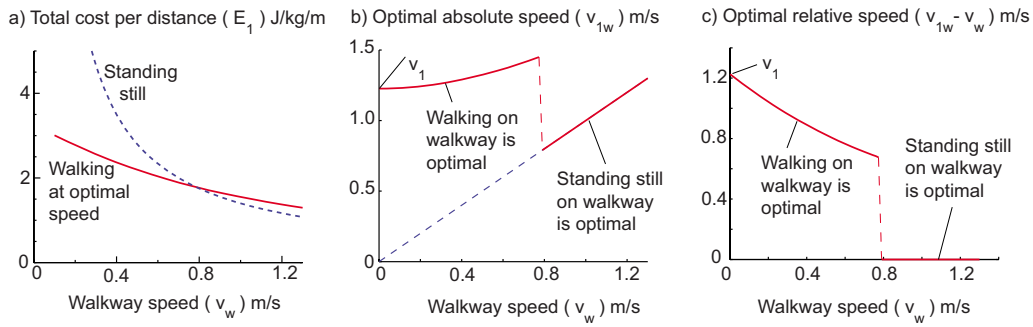


FIG. 2. (Color online) Walking on a moving walkway, minimizing the cost per unit distance traveled. (a) Costs for walking at the optimal walking speed and cost for standing still on the walkway. [(b) and (c)] Walking at the optimal speed [Eqs. (5) and (11)] is better than standing still on the walkway at low walkway speeds; standing still is better at higher walkway speeds. The transition for human walking is at about 0.75 m/s. This figure uses values for  $a_1$  and  $a_2$  from Sec. III B.

### A. Theory 1: Energy minimization

Given a moving walkway traveling at a constant speed  $v_w$ , we might ask what the various energy-optimal speeds are. While walking on a moving walkway is a novel task, something humans are unlikely to have encountered during their evolutionary history, there is some evidence that humans are able to adapt and optimize subject to novel constraints, as noted earlier.<sup>5</sup>

Say, the speed of the person relative to the ground is  $v_a$  (the absolute speed) and the speed relative to the walkway is  $v_r = v_a - v_w$  (the relative speed). The total metabolic rate depends only on what the legs are doing relative to the walkway, so that  $\dot{E}_m = a_1 + a_2 v_r^2 = a_1 + a_2 (v_a - v_w)^2$ .

First, let us try to minimize the total metabolic cost per unit distance traveled relative to the ground. This is the same as walking the length of the moving walkway with the least total expenditure of energy or reaching the end of the walkway with the greatest unused energy.

If the person just rode the walkway standing still, the cost per distance would be  $e_{\text{rest}}/v_w$ . On the other hand, if the person walked with positive relative speed ( $v_r > 0$ ), the total cost per distance would be

$$E_1 = (a_1 + a_2 v_r^2)/v_a = (a_1 + a_2 (v_a - v_w)^2)/v_a. \quad (9)$$

This quantity  $E_1$  is minimized at the absolute velocity  $v_a = v_{1w}$ , given by

$$v_{1w} = \sqrt{\frac{a_1}{a_2} + v_w^2} = \sqrt{v_1^2 + v_w^2} \geq v_1, \quad (10)$$

where  $v_1 = \sqrt{a_1/a_2}$  is the optimal speed when  $v_w = 0$  [reducing to Eq. (5)]. The optimal speed relative to the walkway is then

$$v_{1w} - v_w = \sqrt{v_1^2 + v_w^2} - v_w \leq v_1. \quad (11)$$

This optimal walking speed can be substituted in Eq. (9) to obtain the corresponding cost per distance, plotted in Fig. 2(a). Is this cost bigger or smaller than the cost per distance for standing still, namely,  $e_{\text{rest}}/v_w$ ? Figure 2(a) shows that walking on the walkway at the optimal speed  $v_{1w}$  from Eq. (10) is energetically better than standing still on the walkway until a certain  $v_w$ , after which it is better to ride the walkway standing still, as depicted in Figs. 2(b) and 2(c). That is, on

high-speed walkways,  $E_1$  is minimized by standing still and on low-speed walkways,  $E_1$  is minimized by walking on it according to Eq. (10).

At low walkway speeds, when walking rather than standing still is optimal, this theory predicts that a person would walk with a greater absolute speed than he would on stationary ground [Eq. (10) and Fig. 2(b)]. On the other hand, at such low walkway speeds, this theory predicts that his speed relative to the walkway would be smaller than his optimal speed  $v_1$  on stationary ground [Eq. (11) and Fig. 2(c)]. At higher walkway speeds, when it becomes advantageous to stand still instead of walk, we predict that the absolute velocity of the person (now equal to  $v_w$ ) can be smaller or larger than the optimal speed  $v_1$  on stationary ground [see Fig. 2(b)] depending on the walkway speed  $v_w$ .

Thus far in this section, we have considered the minimization of the total cost per distance  $E_1$ . What if we minimize other energy costs? For instance, using the idealized modern human daily routine described in Sec. III C [see enumerated list just before Eq. (7)], we might ask what the optimal strategy is if someone wished to minimize the total metabolic expenditure over a long-enough time period  $T$ , and they are constrained to travel a certain distance on the walkway. Clearly, in this case, it is impossible to do better than stand still on the walkway. If people in airports were performing exactly this optimization, in the absence of time constraints, we would find no one walking on the walkway, only people riding the walkway standing still.

### B. Theory 2: Resolving sensory conflicts optimally

We now consider a substantially different theory to predict the speed choice on a moving walkway—a theory not based on energy minimization but on the resolution of sensory conflicts. Perhaps, when a human is walking at some preferred speed, he stabilizes around that speed by some feedback mechanism, relying implicitly on estimations of the speed at which he is walking. Such speed estimations might come from integration of various sensory modalities (see, for example, Refs. 42–44). Here, for simplicity, we will lump the various sensory modalities into two: (1) the visual flow information from the eye and (2) information from the legs about what the muscles are doing. On stationary ground,

these two sources of information might be expected to agree about the speed at which the person is walking. However, on a moving walkway, the visual flow estimates the absolute walking speed but the legs estimate the relative walking speed. Most simply, if the eye says that you are walking at  $v_a$  and the legs say you are walking at  $v_r = v_a - v_w$ , the brain might reconcile these conflicting sensory readings by averaging over them. If the reliability of these sensors were about equal, a simple average would be optimal (in a sense to be elaborated below). So the brain might estimate the walking speed to be  $(v_a + v_r)/2 = v_a - v_w/2$ . Then, one might suppose that the human might try to make this best estimate of the walking speed equal to the preferred speed  $v_p$  on stationary ground. That is,

$$\begin{aligned} v_a - v_w/2 &= v_p, \\ \Rightarrow v_a &= v_p + v_w/2 \quad \text{and} \quad v_r = v_1 - v_w/2. \end{aligned} \quad (12)$$

This analysis can be made a little more general by assuming specific probabilistic models for the sensors, allowing for different confidences in the two sensors. I now briefly describe such a more general analysis for the resolution of sensory conflict.

Say that the sensors are both noisy and their readings are normally distributed about the true mean. In particular, say that the probability that the eye makes a velocity reading  $u_E$  when the actual visual flow is  $v_E$  is

$$p_E(u_E) = \frac{1}{\sigma_E \sqrt{2\pi}} \exp\left(-\frac{(u_E - v_E)^2}{2\sigma_E^2}\right). \quad (13)$$

Similarly, assume the probability that the legs infer a velocity  $u_L$  when the speed relative to the ground is  $v_L$  to be

$$p_L(u_L) = \frac{1}{\sigma_L \sqrt{2\pi}} \exp\left(-\frac{(u_L - v_L)^2}{2\sigma_L^2}\right). \quad (14)$$

Now, say the eye makes a velocity reading  $u_E$  and the legs make a velocity reading  $u_L$ . Assuming no sensor integration over time, how might the brain resolve this conflict optimally? A naive brain, not used to walking on moving walkways, is likely to implicitly believe that the true mean of the quantities estimated by the eye and the legs are equal, that is,  $v_E = v_L = \mu$ . Then, the brain may estimate  $\mu$  as that value of the mean that would maximize the probability  $p_E p_L$  of making the sensor readings  $u_E$  and  $u_L$  simultaneously. It can be shown that this *maximum likelihood estimate*<sup>45,46</sup>  $\mu$  of the walking velocity is given by

$$\mu = \left(\frac{1}{\sigma_E^2} + \frac{1}{\sigma_L^2}\right)^{-1} \left(\frac{u_E}{\sigma_E^2} + \frac{u_L}{\sigma_L^2}\right), \quad (15)$$

$$= \frac{\lambda_E u_E + \lambda_L u_L}{\lambda_E + \lambda_L}, \quad (16)$$

where  $\lambda_E = 1/\sigma_E^2$  and  $\lambda_L = 1/\sigma_L^2$  are measures of confidence in the respective sensory modalities.

Equations (13) and (14) imply that the expected sensor reading from the eye will be  $\langle u_E \rangle = v_a$ , the true absolute speed, and the expected sensor reading from the legs will be

$\langle u_L \rangle = v_r = v_a - v_w$ , the true speed relative to the walkway. Using these expectations in Eq. (16), the expected velocity estimate is

$$\langle \mu \rangle = \frac{\lambda_E v_a + \lambda_L (v_a - v_w)}{\lambda_E + \lambda_L}. \quad (17)$$

This is also the most probable walking speed estimate.

If we assume that the human might try to equate the estimated walking speed  $\langle \mu \rangle$  from Eq. (17) to the preferred speed  $v_p$  on stationary ground, we get

$$\langle \mu \rangle = v_p,$$

so that

$$v_a = v_p + \frac{\lambda_L}{\lambda_E + \lambda_L} v_w. \quad (18)$$

$$v_r = v_p - \frac{\lambda_E}{\lambda_E + \lambda_L} v_w. \quad (19)$$

Note that Eqs. (18) and (19) reduce to Eq. (12) when the two sensory modalities are equally reliable (that is,  $\sigma_E = \sigma_L$  and  $\lambda_E = \lambda_L$ ).

Two other simple limits are also interesting. When the brain trusts the eye entirely ( $\sigma_E \ll \sigma_L$  and  $\lambda_E \gg \lambda_L$ ), the absolute velocity on the walkway will be  $v_p$  independent of  $v_w$ . When the brain trusts the legs entirely ( $\sigma_E \gg \sigma_L$  and  $\lambda_E \ll \lambda_L$ ), the relative velocity on the walkway will be equal to  $v_p$  independent of  $v_w$ .

From Eqs. (18) and (19), one would generically predict that the absolute speed  $v_a$  of the walker on the walkway would be faster than his speed  $v_p$  on stationary ground, but his relative speed  $v_r$  would be smaller than his speed on stationary ground. In this qualitative sense, the sensory conflict theory is similar to the energy optimality theory at low walkway speeds. Unlike the energy optimality theory, the above sensory conflict theory would not predict that people would choose to stand still at higher walkway speeds. However, one might perhaps reasonably append to the sensory conflict theory with the rule that when the walkway speed is greater than the preferred speed, the subject will choose to stand still.

### C. Comparison with observations

The predictions of the two theories presented above are testable by observations of people walking on moving walkways. Young<sup>47-49</sup> studied the walking behavior at the San Francisco International airport and the Cleveland Hopkins International airport. He determined the frequency distributions of the speeds used by people walking on moving walkways and people walking next to the moving walkways on stationary ground, bypassing the walkways. The speed distribution for walking on the moving walkway had greater variance than the speed distribution on stationary ground, with a small fraction of people standing still on the moving walkway. Overall, it was found that the distribution of relative speeds on the moving walkway is shifted somewhat to the left of the stationary ground speed distribution.

In particular, at the San Francisco airport, the speeds of the walkways were about  $v_w=0.64 \text{ ms}^{-1}$ . The mean speed for walking on stationary ground, bypassing the walkways, was about  $1.34 \text{ ms}^{-1}$  and the most likely bypass speed (peak of the speed distribution) was about  $1.22 \text{ ms}^{-1}$ . The mean relative speed for those that walked on the walkways was about  $1.03 \text{ ms}^{-1}$  and the most likely relative speed was about  $1.07 \text{ ms}^{-1}$ . Thus, overall the relative speed on the walkway was greater than the bypassing walking speed, but the absolute speed of walking (mean:  $1.03+0.64=1.67 \text{ ms}^{-1}$ ; most likely:  $1.07+0.64=1.71 \text{ ms}^{-1}$ ) on the walkway was greater than the bypassing walking speed. This ordering of the relative and absolute speeds on the walkway with respect to the bypassing speed is in qualitative agreement with both the energetic optimality theory and the sensory conflict theory. What about quantitative agreement?

Walkers in airports tend to carry suitcases and other luggage which are likely to considerably alter the energetics of walking. Their behavior is also possibly affected by issues such as congestion and time constraints.<sup>48</sup> Nevertheless, it may be instructive to compute what a theory that does not consider such effects predicts. If we used  $a_1$  and  $a_2$  from Sec. III B and  $v_w=0.64$ , the energetic optimality theory predicts an absolute velocity [Eq. (10)],

$$v_{1w} = \sqrt{(a_1/a_2) + v_w^2} = 1.39 \text{ ms}^{-1}, \quad (20)$$

which is considerably smaller than the observed mean of  $1.67 \text{ ms}^{-1}$ . If we used the on-ground preferred speed of  $1.34 \text{ ms}^{-1}$  for  $v_1$  in Eq. (10), we obtain a  $v_{1w}$  of about  $1.58 \text{ ms}^{-1}$ , closer to  $1.67 \text{ ms}^{-1}$ . In both cases, it appears that the energetic optimality theory, in the form presented here, underestimates people's walking speeds on moving walkways.

The sensory conflict theory [Eqs. (18) and (19)] has effectively one free parameter  $\lambda_L/(\lambda_L+\lambda_E)$ , and specific quantitative predictions of preferred speeds may not be made without knowing the value of this parameter. The two theories may be distinguished if one performed experiments which measured the dependence of the walking speed as a function of the walkway speed. The sensory conflict theory (in its specific form here) gives a linear dependence of the walking velocity on the walkway speed, whereas the energetic optimality theory gives a nonlinear dependence. Further, if the dependence was found to be linear, consistent with the sensory conflict theory, the data would allow us to estimate the value of  $\lambda_L/(\lambda_L+\lambda_E)$ .

## V. DISCUSSION

This paper has touched on the application of energetic optimality to the prediction of preferred speeds for locomotion on stationary ground and on moving walkways in the absence of explicit speed or time constraints. Humans, however, often operate under explicit time constraints or at least assign an implicit cost for time spent in locomotion (time is money they say). Perhaps reflecting such costs for time, it has been observed that pedestrians in different cities often travel at slightly different speeds, their speeds being correlated with some measure of "the pace of life,"<sup>50,51</sup> the size

and affluence of the various cities. However, it would be interesting to determine the metabolic cost per distance as a function of speed for individuals drawn from the various cities to see how much of the variation in walking speeds can be explained by minimization of the metabolic cost—people from different countries might have different mean values of height, weight, and other physiological parameters that determine the metabolic cost of walking. Such a study appears to have not been performed.

While the Ralston experiment of asking the subject to walk most naturally produces a reasonably narrow range of speeds, humans in their daily life use a broad range of speeds. Such a broad distribution can partly be explained by assuming a distribution of time constraints and motivations, by assuming noisy sensors and muscles, by noting that different humans are characterized by different gross physical parameters, and perhaps by assuming that a human, at any given instant, is characterized by a number of internal physiological state variables (for example, perhaps one reflecting the fatigue level of the human), whose values are different for different humans at any given moment and are different at different times for the same human.

The energetic optimality theory for walking on the moving walkway implicitly assumed that the brain is able to independently and accurately measure the absolute speed (perhaps using the visual flow) and the relative speed or the energy rate (perhaps from feedback from the leg muscles). This was a simplifying assumption. Building on the two theories here, it would be natural and appropriate to consider a hybrid theory that incorporates aspects of both—that is, minimize energy while optimally reconciling noisy information from the different sensors. Detailed discussion of this theory is beyond the scope of this paper, but let us briefly ponder the likely consequences of such a hybrid theory. First, this hybrid theory may predict that a small fraction of people will stand on the walkway even at lower walkway speeds—because the noisy and conflicting sensors will now make it impossible to have 100% confidence on which strategy is better, standing or walking. Second, it may also remove the underestimation of the observed walkway speeds by the deterministic energy optimality (especially given that there will now be a free parameter in the theory, relating to the relative confidences in the two sensory modalities).

One reason to use moving walkways in airports is to reduce the passenger transit times between airport terminals.<sup>52</sup> But standing still on the walkway instead of walking on the moving walkway or bypassing the walkway may *reduce* the overall transport speed compared to when there is no walkway, thus increasing the transit times.<sup>47,48</sup> One might perhaps draw a parallel (albeit an inexact one) with the so-called Braess paradox,<sup>53,54</sup> in which the addition of extra links to a traffic network sometimes increases congestion because of the way in which the extra links affect the ensemble behavior of selfish individuals, converging to a worse-than-before noncooperative Nash equilibrium.

The moving walkway analysis described here also applies to walking experiments in which the visual flow is externally altered by using a wearable virtual reality device or by projecting moving images on the walls or on a

screen.<sup>42,44,55–57</sup> For instance, Mohler *et al.*<sup>42</sup> recently found that the preferred speed decreased with increased visual flow speeds, again consistent with the two theories here. Mohler *et al.*<sup>42</sup> also found that the walk-run transition speeds also decreased with increased visual flow speed. In related work, it has also been found that a number of animals also show considerable locomotor responses to artificially induced visual flow.<sup>58–60</sup>

The sensory conflict theory presented here is clearly not the most general possible. For instance, it does not account for the possibility that when the sensory conflict between the eyes and the legs becomes high enough, the brain may choose one or another sensory modality instead of reconciling the two smoothly as here (see in Ref. 61 for references). Closely related, in the presence of a sensory conflict, it may be that the brain relearns the appropriate weighting for each sensory modality over time (e.g., Ref. 62), resulting perhaps in a preferred speed that slowly changes over time. Finally, almost all of the discussion here assumes the lack of conscious cognitive reasoning, sound or otherwise, in making decisions about walking speeds. For instance, one might as well attribute the reduced relative speed or standing on a walkway to the person explicitly reasoning about the utility of the walkway<sup>47</sup> and trying to reduce exertion as the person consciously perceives it.

## VI. CONCLUSION

The central theme of this paper has been the use of energetic optimality and related ideas to make predictions about and explain animal behavior, in particular, explaining the speeds they use to locomote. The eventual goal of such research is obtaining a good-enough theory capable of predicting how humans would behave in novel locomotor situations and how, for instance, humans would respond to novel surgical procedures or prosthetic devices.

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